OUNCES OF COFFEE TASK
SEVENTH GRADE LESSON GUIDE

LESSON OVERVIEW:

Students will be presented with the Ounces of Coffee problem. They will be asked to determine whether or not the there is a proportional relationship between the ounces of coffee to the price. Students then will be asked to find the unit price and explain in writing what the unit price means in the context of the problem. Finally, students will explain why it is helpful to determine if the relationship between the amount of coffee and price is proportional.

Students should be able to see the proportional relationship between the ounces of coffee and the price. They should be able to find the unit price and to see that the cost per ounce is the same.

Before working on the Ounces of Coffee task, students will do a Warm-Up task identifying ratios and the appropriate representations of unit rates. Students will be allowed 15-20 minutes to engage in and discuss the Warm-Up task. At least 2-3 periods will be allotted for students to explore and share their work.

COMMON CORE STATE STANDARDS:

- **6.RP.1** Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.
- **6.RP.2** Understand the concept of a unit rate *a*/*b* associated with a ratio *a*:*b* with *b* ≠ 0, and use rate language in the context of a ratio relationship.
- **6.RP.3** Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
  a. Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
  b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*
  c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.
  d. Use ratio reasoning to convert measurement units to manipulate and transform units appropriately when multiplying or dividing quantities.
- **7.RP.1** Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction 1/2/1/4 miles per hour, equivalently 2 miles per hour.*
- **7.RP.2** Recognize and represent proportional relationships between quantities.
  d. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
  e. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

f. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn*.

2. Explain what a point *(x, y)* on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r)* where *r* is the unit rate.

- **7.RP.3** Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*
- **8.EE.5** Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.
- **8.EE.6** Use similar triangles to explain why the slope *m* is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at *b*.

DRIVING QUESTION:	NCTM ESSENTIAL UNDERSTANDINGS[3]:	SKILLS DEVELOPED:
• What are different types of ratios? • How can ratios be used to make comparisons? • How are ratios related to fractions?	1. Reasoning with ratios involves attending to and coordinating two quantities 2. A ratio is a multiplicative comparison of two quantities, or it is a joining of two quantities in a composed unit. 3. Forming a ratio as a measure of a real world attribute involves isolating that attribute from other attributes and understanding the effect of changing each quantity on the attribute of interest. 4. Ratios can be meaningfully reinterpreted as quotients. 5. Proportional reasoning is complex and involves understanding that: • Equivalent ratios can be created by iterating and/or partitioning a composed unit; • If one quantity in a ratio is multiplied or divided by a particular factor, then the other quantity must be multiplied or divided by the same factor to maintain the proportional relationship.	• Use different representations to form ratios and make comparisons with ratios. • Form equivalent ratios and use equivalent ratios to solve problems. • Find the Unit Rate.

MATERIALS:	GROUPING:
Warm up task, Ounces of Coffee sheet, calculators, Smartboard	Students will work alone and then in groups of three to four.

[3] NCTM (2010) Developing Essential Understandings of Ratios, Proportions & Proportional Reasoning: Grades 6 -8.

SET-UP

Instructions to Students: Students will discuss their understanding of ratio, unit rate, rate and proportion. A student will be told to read the problem while others follow along silently. Explain to students that they will have to justify their solutions and explain their reasoning. Students will be told to work alone for 7 to 10 minutes and then in small groups.

EXPLORE PHASE: Supporting Students' Exploration of the Mathematical Ideas

Private Think Time: Allow students to work individually for 3-5 minutes without intervening, though you might want to circulate quickly to get an idea of the strategies that they are using.

Small-Group Work: After 3-5 minutes, ask students to work with their partner or in their small groups. As students are working, circulate around the room. Be persistent in:

- asking questions related to the mathematical ideas, problem-solving strategies, and connections between representations.
- asking students to explain their thinking and reasoning.
- asking students to explain in their own words, and build onto, what other students have said.

As you circulate, identify solution paths that you will have groups share during the Share, Discuss, Analyze Phase, and decide on the sequence that you would like for them to be shared. Give groups a "heads up" that you will be asking them to come to the front of the room. If a document reader is not available, give selected groups an OVH transparency or chart paper to write their solution on.

Possible Solution Paths	Possible Assessing and Advancing Questions
If a group is unable to start: Focus students on the table. 5. *What does the task ask us to figure out?* 6. *What is being compared?* 7. *What are the items on the table being compared?* *We are comparing ounces of coffee to price in dollars.*	Assessing • What are we trying to figure out in this problem? • What can you tell me about the ounces of coffee and the price in dollars?
oz. oz. oz. oz. oz. oz. .40 .40 .40 .40 .40 .40	Assessing Questions • Tell about your work. • How did you figure out that each ounce would cost 40¢? • How did you know that each ounce of coffee would cost forty cents? Advancing Questions • Is there another way to show the relationship between the amount of coffee and the price?

0.40
6 |2.40
2.40

0.40
8|3.20
3.20

0.40
16|6.40

Assessing Questions

- Tell us about your work.
- Why are you dividing the price by the ounces?
- What does the .40 tell us? What do we call it?
- How do you know that dividing the ounces by the dollars will give you the unit rate?

Advancing Questions (Not all of these would be asked at the same time.)

- How does knowing the unit price benefit you?
- How can you represent the quantities from the table in a ratio? Can you represent the ratio in fraction notation?
- How can you compare the ratios? Are they the same or equivalent?
- Is there a proportional relationship?

1	*.40*
2	*.80*
3	*1.20*
4	*1.60*
5	*2.00*
6	*2.40*
8	*3.20*

Assessing Questions

- Tell me about your table and what you noticed. Do you see a pattern?
- What patterns do you see in your table?

Advancing Questions

- If students have not noticed a pattern then, tell me about the pattern in the table?
- What does the pattern tell you?
- So if you have 32 ounces of coffee, how much will that cost?
- Is there a proportional relationship between the ounces of coffee and the cost? Why or why not?
- Will this method for solving for proportion always work? How do you know?
- Can you think of a proportion that this method of solving will not work with?

Possible Errors and Misconceptions	Possible Questions to Address Errors and Misconceptions
Students may ignore the decimal point in the $2.40, $3.20 and $6.40.	Assessing Questions • What is being compared? • Where did you get the 240, 320, and 640 from? Or devils advocate: Wow is that 240 dollars? • How is it the same as the quantities in the table? Advancing Questions • What are the quantities being compared in this problem? • How are you going to use ratios to help you see if there is a proportional relationship?

SHARE DISCUSS ANALYZE PHASE: Deepening and Connecting Mathematical Understanding

General Considerations:

- Orchestrate the class discussion so that it builds on, extends, and connects the thinking and reasoning of students
- Sequence the solution paths so that you will be able to press students to make comparisons and connections across, and between, the various solution paths.

Possible Sequence of Solution Paths	Possible Questions and *Possible Student Responses*
A focus on Pattern Finding and Describing the Proportional Relationship *ORTIZ* *1 — .40* *2 — .80* *3 — 1.20* *4 — 1.60* *5 — 2.00* *6 — 2.40* *8 — 3.20* *16 — 6.40* I looked at the possible ratio as a fraction and simplified it to its lowest terms. I found that the ounces of coffee to price in dollars were a ratio of 1 to .40 for each given quantity.	• Tell us about your work. • Do you see a pattern? • What patterns do you see in your table? • What made you create this table? • What does the pattern tell you? • Is there a proportional relationship here? What is it? • What was the method that this group used to figure out if there was a proportional relationship? Will this method for solving for proportion always work? How do you know? • So, if you have 32 ounces of coffee, how much will that cost? • Can you think of a proportion that this method of solving will not work with?

A) There is a proportional relationship between ounces of coffee and price in dollars because when you divide ounces of coffee to the price in dollars, it gives you 1/.40 or 1 divided by .40 and this is for each quantity given. For example, 6/2.40 = 1/.40 and 8/3.20= 1/.40 and 16/6.40 = 1/.40 so the relationship is proportional. B) 1/.40 is the unit rate for this problem. When I simplified 6 over \$2.40; 8 over \$3.20 and 16 over \$6.40, they all resulted in 1 over .40. C) For every ounce of coffee the price is 40¢. D) The reason why Julia has to find out if it is proportional is the lowest price per ounce would be the best bag to buy. If it is proportional then Julia will know that it is okay to purchase any bag because the coffee price will always remain the same per ounce of coffee. If not, Julia will need to find the lowest price per ounce of coffee.	
Miraj A Focus on using ratios to see if the different price and amount are proportional. Since we have three different sized bags and three different prices, I will use ratios to compare the amount of coffee expressed in ounces to the price in dollars. Based on the table given, I will write the following ratios: \$2.40 : 6 or \$2.40 to 6 or \$2.40/6 \$3.20 : 8 or \$3.20 to 8 or \$3.20/8 \$6.40 :16 or \$6.40 to 16 or \$6.40/16 I am trying to see if there is any proportional relationship between the amount of coffee and the price. To do that, I had to compare to see if the ratios I created are the same. To see if the ratios are proportional, I cross multiplied and I found out that the first two ratios are in a proportional. The evidence for that is the equation $\frac{\$2.40}{6} = \frac{\$3.20}{8} = \frac{\$6.40}{16}$	Explain your group's solution. • How did you find out whether or not the ratios were the same or proportional? • How did you know that they were in a proportional relationship? • If you had \$19.20 how many ounces of coffee can you buy? • How can you represent this information in another way?

B) The unit rate will be determined by one of the ratios since all of the ratios are the same I will take $2.40 divided by 6 equals $.40 or forty cents. C) The unit rate means the price for coffee for each ounce in each bag is the same, forty cents. D) It is important for Julia to know that the amount of coffee and the price of the coffee is proportional so she can calculate how much money she will need for the new bag of coffee.	
Boatright: A Focus on using quotient to get the price of Ounces/Price in Dollars 6/2.40 ÷ 6/6 = 1/.40 Ounces/Price in Dollars 8/3.20 ÷ 8/8 = 1/.40 Ounces/Price in Dollars 16/6.40 ÷ 16/16 = 1/.40 I looked at the possible ratio as fractions and simplified it to its lowest terms. I found that the ounces of coffee to price in dollars were a ratio of 1 to .40 for each given quantity. The unit rate is one ounce per $ 0.40 a. Yes, the proportion is equal to 1/.40, for each quantity given. Therefore, it is proportional. b. 1/.40, is the unit rate because when simplified the ratio is one to 40 hundredths. c. Ounce of coffee/price of coffee is 1/.40. This means that one-ounce cost 40 cents. d. It is helpful for Julia to find the unit price because this way she assures herself that each coffee package has the same cost per ounce.	Explain your group's solution. • What is a ratio? • What is a proportion? • Why are you simplifying? • What is a unit rate? What does it mean in this context? • Can a unit rate be simplified? • Can a unit rate be negative?

CLOSURE
Quick Write: Choose one of the questions below depending on your students' understanding. • **WHEN SOMETHING IS CHANGING PROPORTIONALLY, WHAT INFORMATION CAN WE GET FROM THE RATIO TO DESCRIBE THE CHANGE?** • **What does it mean if there is a proportional relationship? Refer to the coffee problem in your explanation. (I wonder if this will permit you to see if they refer to the ratio.)**
Possible Assessment: • Continue to provide similar problems in which various solution paths can be used.
Homework: • Jose and Russell jogging problem, weight on the moon problem and the light bulb problem.

OUNCES OF COFFEE

Name ________________________ Date ____________________

Julia made observations about selling price of a new coffee that sold in three different sized bags. She recorded those observations in the following table:

Ounces of Coffee	6	8	16
Price in Dollars	**$2.40**	**$3.20**	**$6.40**

a) Is there a proportional relationship between the amount of coffee and the price? Why or why not?

____
____
____
____

b) Find the unit rates associated with the problem.

____
____
____

c) Explain in writing what the unit rates mean in the context of this problem.

____
____
____

d) Explain in writing why is it helpful for Julia to determine if the relationship between the amount of coffee and the price is proportional before she buys a new bag of coffee.

____
____
____